QUIZ 3 REVIEW

Chapter Concepts

6.1-2 Eigenvalues + Eigenvectors λ, X

6.3 Differential Equations $\frac{du}{dt} = Au$ and e^{At}

6.4 Symmetric Matrices have real eigenvalues
$A = A^T = Q\Lambda Q^T$

6.5 Positive Definite matrices

6.6 Similar Matrices have the same eigenvalues
$B = M^{-1}AM$, $B^k = M^{-1}A^kM$

6.7 Singular Value Decomposition
$A = U\Sigma V^T$

PROBLEM 1

$$\frac{du}{dt} = Au = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} u$$

General Form of Soln:

$$u(t) = C_1 e^{\lambda_1 t} X_1 + \dots + C_3 e^{\lambda_3 t} X_3$$

A is singular so $\underline{\lambda_1 = 0}$

If A is diagonalizable then
$$e^{At} = S e^{\Lambda t} S^{-1}$$

$$|A - \lambda I| = \begin{vmatrix} -\lambda & -i & 0 \\ i & -\lambda & -1 \\ 0 & 1 & -\lambda \end{vmatrix} = \lambda^3 + 2\lambda = 0$$

$$\lambda(\lambda^2 + 2) = 0$$

EIGENVALS

$\underline{\lambda_2 = \sqrt{2}i}$, $\underline{\lambda_3 = -\sqrt{2}\, i}$

PROBLEM 2

$\lambda_1 = 0,\ \lambda_2 = c,\ \lambda_3 = 2$

$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $x_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ $x_3 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$

Given a matrix A (3×3) w/ these eigenvalues / vectors

* note, x's are orthogonal!

(a) Is A diagonalizable? (or for which c is it?)

- A matrix is diagonalizable when its eigenvectors are independent
- YES, diagonalizable for all c (lecture 22)

(b) is A symmetric?

- A matrix is symmetric when $A = A^T$, and (lecture 25) the eigenvalues are real, eigenvectors are perpendicular
- YES for real c

(c) Is A positive definite?

- A must be symmetric which it is.. (lecture 27)
- <u>No</u> $\lambda_1 = 0$ and we need $\lambda > 0$

(d) Is A a Markov Matrix?

- No, $\lambda_3 > 1$ and we need $\lambda < 1$ (lecture 24)

(e) Is $A/2$ a projection matrix?

- The λ's of a projection are $\lambda = 0, 1$ bc $\underline{P^2 = P}$ so $\underline{\lambda^2 = \lambda}$
- Yes for $c = 0$ or $c = 2$ else NO

PROBLEM 3 SINGULAR VALUE DECOMPOSITION

For every matrix $\rightarrow$ $A = (\text{orthog})(\text{diag})(\text{orthog}) = U\Sigma V^T$

For symmetric matrices $U = V$ $\rightarrow$ $A^TA = (V\Sigma^T U)(U\Sigma V^T) = V(\Sigma^T\Sigma)V^T$

The diagonals of Σ are $\sigma_i = \sqrt{\text{eigenvals of } A^TA}$

PROBLEM 4

Given matrix A is symmetric and orthogonal

(a) eigenvalues

Symmetric $\rightarrow$ λ is real

Orthogonal $\rightarrow$ $|\lambda| = 1$ $\leftarrow$ $\begin{cases} Qx = \lambda x \\ \|x\| = |\lambda|\|x\| \rightarrow |\lambda| = 1 \end{cases}$

$\therefore \lambda = 1 \text{ or } -1$

(b) T/F A is Pos Def? Not necessarily

(c) T/F No repeated eigenvalues? May have repeated λ

(d) T/F A is diagonalizable? YES true bc symmetric

(e) T/F A is non-singular? TRUE

(f) Show $\frac{1}{2}(A+I)$ is a projection matrix

A projection matrix P is symmetric and $P^2 = P$

$$\left[\tfrac{1}{2}(A+I)\right]^2 = \tfrac{1}{4}(A^2 + 2AI + I) \overset{?}{=} \tfrac{1}{2}(A+I)$$

<u>Note</u> $A = A^T = A^{-1}$ bc A is symm, orthog.

$$\therefore AA = AA^{-1} = I$$

$$\tfrac{1}{4}(2AI + 2I) = \tfrac{1}{2}(A+I)$$ YES, Proj. Matrix