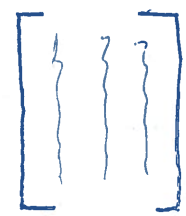
Review Practice Problems

Q: U is 5×3 w/ 3 pivots, $r=3$



- What is the null space?

  → Since full rank, ie #pivots = # cols the cols are independent and no combinations of cols is zero vector except trivial solution $N(U)=0$

- Given $B=\begin{bmatrix}U\\2U\end{bmatrix}$ what is the rank and echelon form?

  → echelon form ⇒ $\begin{bmatrix}U\\0\end{bmatrix}$

  Now do $\underset{10\times6}{C}=\begin{bmatrix}U & U\\U & 0\end{bmatrix} \to \begin{bmatrix}U & U\\0 & -U\end{bmatrix} \to \begin{bmatrix}U & 0\\0 & -U\end{bmatrix}$

  $\text{rank}(B)=3$, $\text{rank}(C)=6$

  $\text{Dim}(\text{Null}(C^T)) = 10\text{ cols} - 6\text{ (rank of C)} = 4$

Q: $Ax=\begin{bmatrix}2\\4\\2\end{bmatrix}$, $x=\begin{bmatrix}2\\0\\0\end{bmatrix}+c\begin{bmatrix}1\\1\\0\end{bmatrix}+d\begin{bmatrix}0\\0\\1\end{bmatrix}$

- What is Dim(row space(A))?

  We know A is 3×3, $\text{rank}(A)=1$ bc $\dim(N(A))=2$

- What does A look like?

  $$A=\begin{bmatrix}1 & -1 & 0\\2 & -2 & 0\\1 & -1 & 0\end{bmatrix}$$

  use nullspace vectors

  $x=[1,1,1]$ $[0,0,1]$

$Ax = b$ can be solved if: (solvable if b in col space of A)

b has the form $b = c\begin{bmatrix}1\\2\\1\end{bmatrix}$

Q: If A is $n\times n$ and $N(A) = 0$, what is $N(A^T)$?

$\rightarrow$ if A is square then $N(A^T) = 0$ as well

Q: A system of n equations and n unknowns is solvable for every R-hand side if the columns are independent. T or F?

$\rightarrow$ $n\times n$ matrix w/ independent cols ($\text{rank} = n$) is $Ax = b$ always solvable? YES full rank, invertible

Q: $$\underset{3\times4}{B} = \underbrace{\overset{C}{\begin{bmatrix}1&1&0\\0&1&0\\1&0&1\end{bmatrix}}}_{3\times3}\underbrace{\overset{D}{\begin{bmatrix}1&0&-1&2\\0&1&1&-1\\0&0&0&0\end{bmatrix}}}_{3\times4}$$

- Give a basis of the null space of B

B is 3×4 so null space vector is $4\times n$ ie $N(B) \subseteq R^4$

Note, matrix C above is square and invertible, so $N(CD) = N(D)$ if C is invertible

D has 2 pivots in first 2 cols, last 2 cols are free

$$N(B) = \begin{bmatrix}+1\\-1\\1\\0\end{bmatrix}, \begin{bmatrix}-2\\1\\0\\1\end{bmatrix}$$

Note, I can use $F = \begin{bmatrix}-1&2\\1&-1\end{bmatrix}$ from D and reverse signs complete soln to $Bx = \begin{bmatrix}1\\0\\1\end{bmatrix}$

$$x_p + x_n = \begin{bmatrix}1\\0\\0\\0\end{bmatrix} + c\begin{bmatrix}1\\-1\\1\\0\end{bmatrix} + d\begin{bmatrix}-2\\1\\0\\1\end{bmatrix}$$

Q: If A,B have same 4 subspaces (null, col, row
then is $A = cb$, A is a multiple of B?

→ False