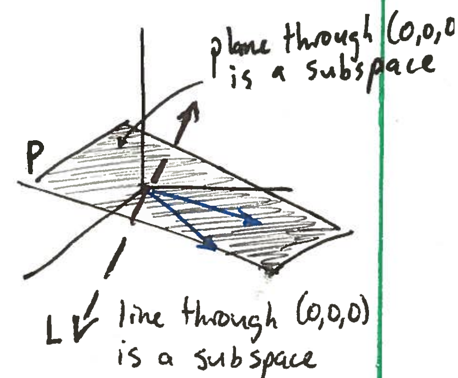
Vector Spaces and Subspaces



Vector Space requirements:

if V and W are in the space,
all comb's cV + dW are in the space

Subspaces:

The union of 2 subspaces is not a subspace. $P \cup L$ not a subspace

The intersect of 2 subspaces is a subspace. $P \cap L = (0,0,0)$

Column Space of A

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix}$$

- Col space of A is a subspace of R^4
- col space of A is all linear comb. of columns of A

* combinations of col's of A do not fill entire space, R^4
  ↳ not always a soln to $Ax = b$
  ↳ 4 equations and only 3 unknowns

$$Ax = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

↑ Solution vector

Which b's allow the system to be solved?

I can solve $Ax = b$ exactly when b is in column space $C(A)$.
The col-space contains all vectors A times any x. That is, solvable when b is a combination of cols and not solvable when not comb. of cols.

3^{rd} column of A contributes nothing, is combination of cols 1,2
So A is a 2D subspace of $\mathbb{R}^4$

Nullspace

The nullspace of A contains all solutions (X) to $Ax = 0$.

The nullspace $(x = (x_1, x_2, x_3))$ is in R^3 (# cols of A).

$$Ax = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Nullspace $N(A)$ contains solutions to $Ax = 0$,

$$x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \ldots \underbrace{c \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}}_{\text{line in } R^3}$$

Check that solns to $Ax = 0$ always give a subspace.

↳ if $Av = 0$ and $Aw = 0$ then $A(v + w) = 0$

This material is covered in book section 3.2