Poynting Vector, Oscillating Charges, Polarization

Electric field density energy

$$u_E = \frac{1}{2}\varepsilon_0 E^2 \quad [J/m^3]$$

Magnetic field density energy

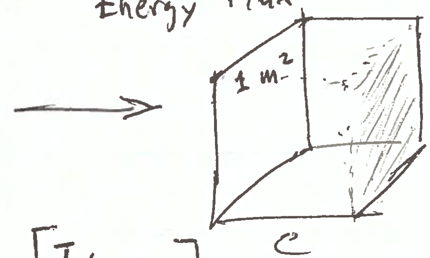
$$u_B = \frac{1}{2\mu_0} B^2 \quad [J/m^3]$$

$$= \frac{1}{2\mu_0}\frac{E^2}{c^2} = \frac{1}{2}\varepsilon_0 E^2$$

Total energy density

$$u_{total} = u_E + u_B = \varepsilon_0 E^2$$
$$= \varepsilon_0 EBc$$

Energy Flux



Volume $= c\,m^3$

Energy flux density...

$$u_{total}\, c = \varepsilon_0 EBc^2 = \frac{EB}{\mu_0} \quad [J/m^2\,sec]$$

↑ Poynting vector

POYNTING VECTOR

$$\vec{S} = \vec{E} \times \vec{B} / \mu_0 \quad \left[\frac{W}{m^2}\right]$$

"the energy transfer per unit area per unit time"

~Energy Flux $\vec{S} = \vec{E} \times \vec{H}$

Time Averaged Value of Poynting Vector

$$\langle S \rangle = \frac{1}{2} E_0 B_0 / \mu_0$$
$$= \frac{1}{2}\frac{E_0^2}{\mu_0 c}$$

"the instantaneous power flow due to instantaneous E/M fields"

Example

Plane E/M wave

$E_0 = 100$ V/m

$$\langle S \rangle = \frac{1}{2}\frac{100^2}{\mu_0 c} = 13 \text{ W/m}^2$$

(Poynting Vector independent of frequency f, ω)

Now, $E_0 = 1{,}000$ V/m

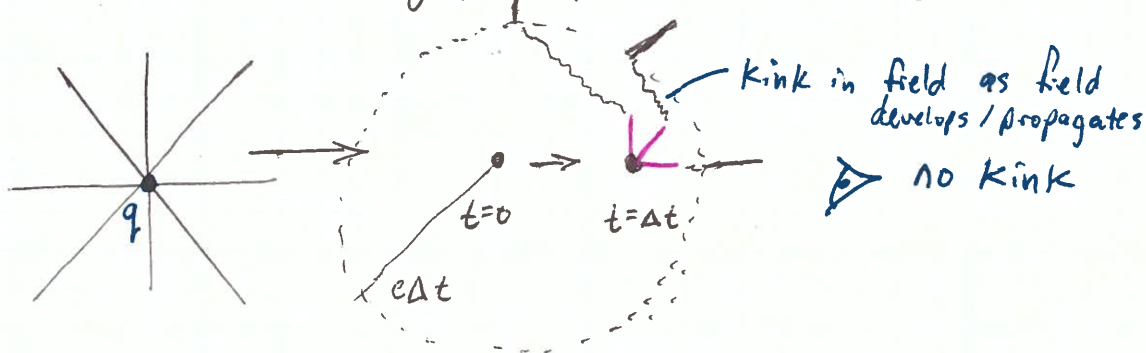
so... $\langle S \rangle = 1.3$ kW/m²

note S scales as E^2

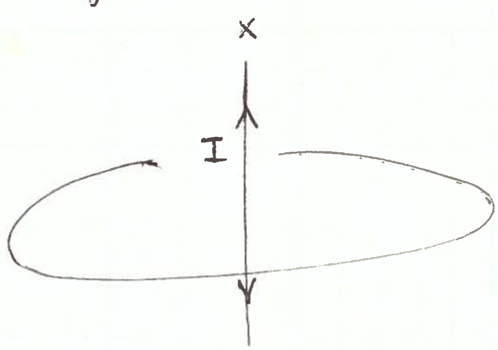
Note, plane waves exist at all time in all of space (planes are infinite in math...)

How is an EM wave produced?

Begin w/ stationary charge, accelerate for time Δt

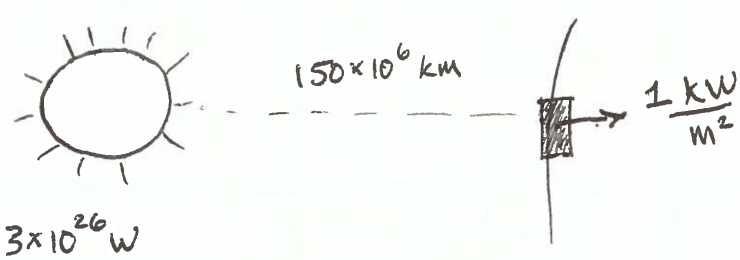


Charges on a wire, antenna



Plane wave solution is not very realistic

Sun's energy at Earth, Poynting vector and E-field



$$\langle S \rangle = \frac{1}{2} \frac{E_0^2}{\mu_0 c}$$

$$\downarrow$$

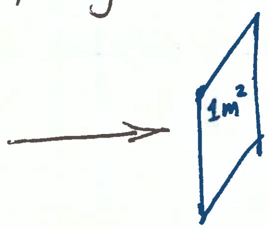
$$1{,}000 = \frac{1}{2} E_0^2 / \mu_0 c$$

$$\underline{E_0 \simeq 870 \ \text{V/m}}$$

Photons are individual packets (~ bullets) that carries energy and so it has momentum.

$$p = \frac{\text{energy of 1 photon}}{c}$$

Poynting Vector and momentum



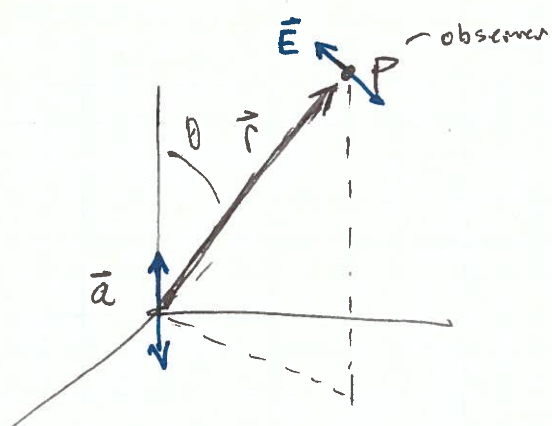
$$\frac{S}{c} = \frac{\text{energy}}{c \cdot m^2\ \text{sec}} = \text{Pressure}$$

So... $\frac{\langle S \rangle}{c} \propto$ = Radiation Pressure

Comets have 2 tails...

one is white due to sun's radiation pressure

other is blue due to solar wind



$\vec{a}, \vec{r}, \vec{E}$ in same plane

oscillating charge is accelerated

We know, $\vec{E} \perp \vec{r}$, $E \propto q \frac{a \sin\theta}{r}$

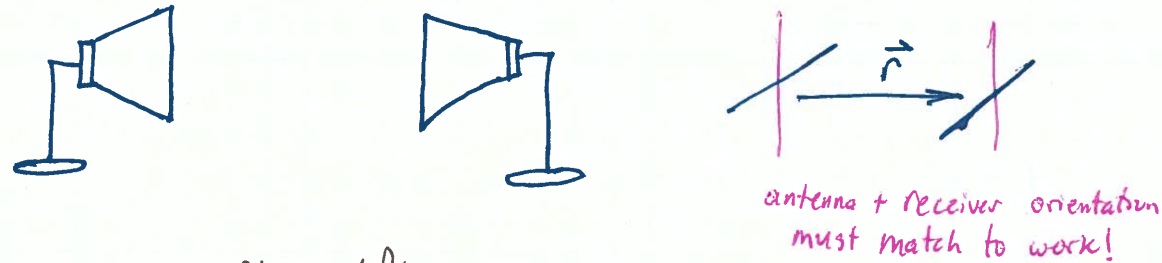
so, $S \propto q^2 \frac{a^2}{r^2} \sin^2\theta$ (conservation of energy)

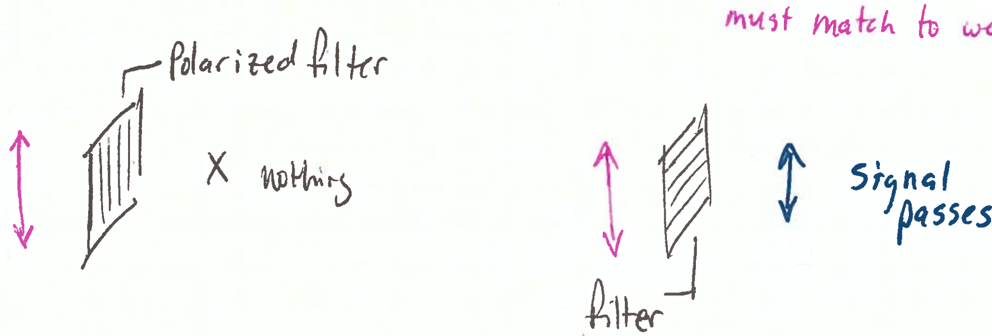
↑ Poynting Vector

DEMO — Linear Polarization

transmitter transmits at 10 GHz ($\lambda = 3cm$)

audio modulated ~ 1 kHz





DEMO

75 MHz ($\lambda = 4m$)