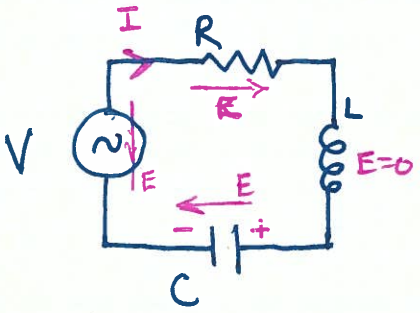
Driven RLC circuits, metal detectors



$V = V_0 \cos(\omega t)$

Recall, KVL does not really hold...
[that is, $\oint E \cdot dl \neq 0$]

$$IR + 0 + V_C - V_0 \cos\omega t = -L\frac{dI}{dt}$$

we know $I = \frac{dq}{dt}$ .... and $V_C = Q/C$

$$\boxed{L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = V_0\cos(\omega t)}$$

Solution to RLC Circuit

Solution for Current in RLC (steady state)

$$\boxed{I = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}\cos(\omega t - \phi)}, \qquad \tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

REACTANCE $X = \omega L - \frac{1}{\omega C}$

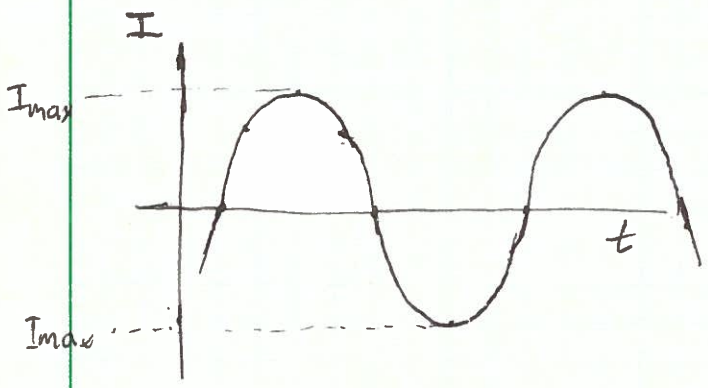
IMPEDANCE $Z = \sqrt{R^2 + X^2}$

$\phi > 0$ means current lags voltage (inductor)

$\phi < 0$ means current leads voltage (capacitor)

Current is $I = I_{max} \cdot \cos(\omega t - \phi)$

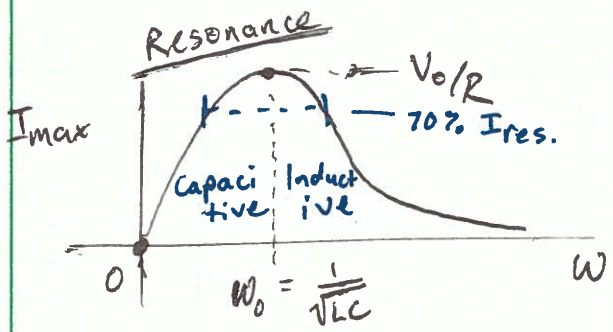
Resonance occurs when I_{max} is greatest, when $\omega L = \frac{1}{\omega C}$



Fix RLC but change driving frequency ω...

[1] $\omega \approx 0$ [DC], $Z \to \infty$, $I_{max} \to 0$

(capacitor charges and acts as an open circuit, does not pass I)



[2] $\omega \to \infty$, $Z \to \infty$, $I_{max} \to 0$

(due to inductor, self inductance dominates)

$$\boxed{\Delta\omega = R/L}$$

$$\boxed{Q = \frac{\omega_0}{\Delta\omega} = \frac{1}{\sqrt{LC}} \cdot \frac{L}{R}}$$

$$\boxed{Q = \frac{1}{R}\sqrt{L/C}}$$ Quality factor

$\Delta\omega$ at 70% of I_{max} is width at half power

DEMO: 200 W lightbulb in RLC circuits

$f = 60$ Hz $(\omega = 377$ Hz$)$

$V = 110\sqrt{2}\cos(\omega t)$

$R = 60\,\Omega$ hot 200 W

$L = 0.1$ H, $C = 8\,\mu$F

→ So.... $Z = 300\,\Omega$

$\omega L = 38\,\Omega$, $\frac{1}{\omega C} = 332\,\Omega$

and $\omega_0 = \frac{1}{\sqrt{LC}} = 1120$ Hz

(note, $\omega = 377_{Hz} << \omega_0 = 1120$ Hz so capacitance dominates!)

$$I_{max} = \frac{V_0}{Z} = \frac{110\sqrt{2}}{300} \approx \frac{1}{2}\text{A}$$

Power in bulb $= \frac{1}{2} I_{max}^2 R = \frac{1}{2}(.5)^2(60) = 7.5$ W

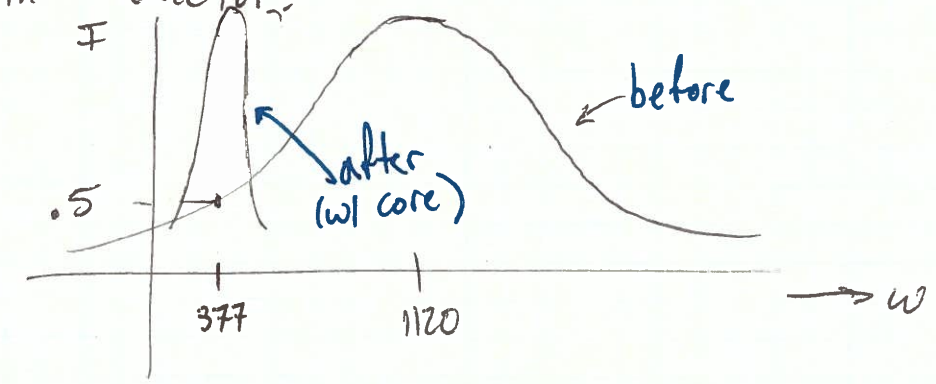
too low power to turn on!

(way below resonance)

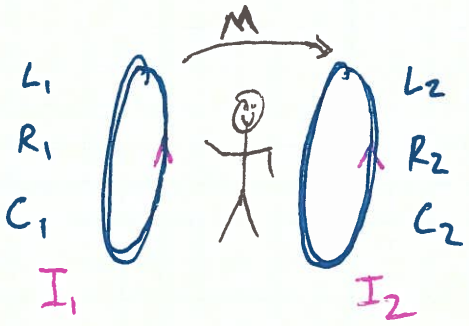
Can we design the system better to light up the bulb? YES, change L, C to shift resonance!

If we shift resonance frequency down to $\omega_0 = 377$ Hz then resonance frequency equals driving frequency ($\omega = \omega_0$) and we are in resonance! (I_{max} is greatest)

Lewin chooses to increase L by inserting iron core in inductor!

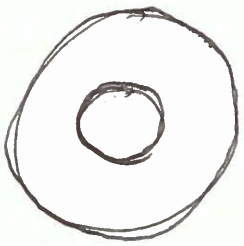


Metal Dector



Mutual Inductance, M

Resonance



Ground Search coil