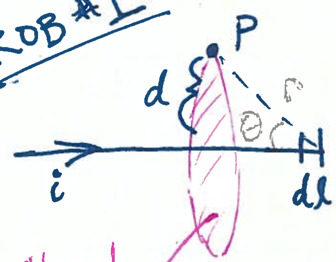
Exam 2 Review

Biot-Savart: $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I}{r^2}(d\vec{l} \times \hat{r})$, **Ampère**: $\oint B \cdot dl = \mu_0\left(I + \varepsilon_0 \frac{d}{dt}\Phi_E\right)$

PROB #1



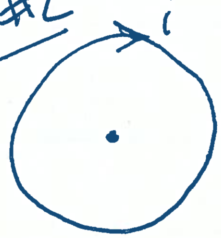
Closed surface for Ampère

Find magnetic field at P due to current i in wire...
(not a good problem for Biot-Savart)

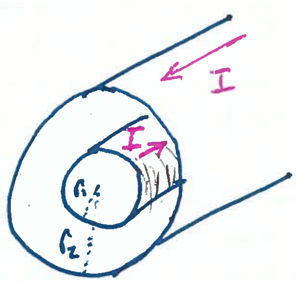
$B \cdot 2\pi d = \mu_0 I$

$\boxed{B = \frac{\mu_0 I}{2\pi d}}$ by Ampère (good when we have cylindrical symmetry)

PROB #2

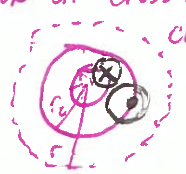


Find B-field at center of current loop
(now let's use Biot-Savart)
(Ampère's Law is no good)



Current I running on inner conductor and returning on outer conductor, find magnetic field everywhere in space

look at cross section, choose surface $r > r_2$



Ampère:

$r > r_2$: $B \cdot 2\pi r = \mu_0 \cdot 0$

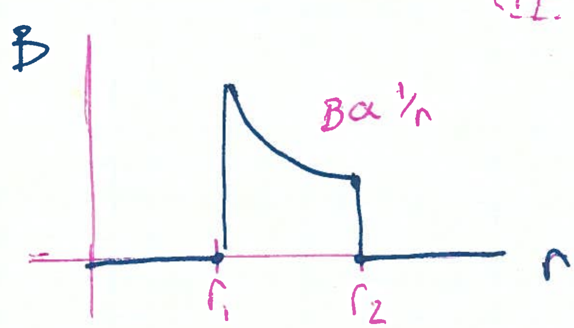
$\boxed{B = 0}$

$r_1 < r < r_2$: $B \cdot 2\pi r = \mu_0 I$

$\boxed{B = \mu_0 I / 2\pi r}$

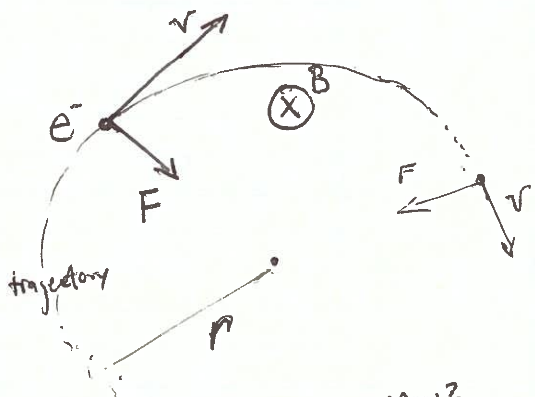
$r < r_1$: $B \cdot 2\pi r = \mu_0$

$\boxed{B = 0}$



Lorentz Force: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Force due to charge q moving at velocity v in a magnetic field B where there is an electric field E.



An electron in a magnetic field (into board) moves in orbit like so.
Force vector points to center of circle.
Speed does not change, force does no work.

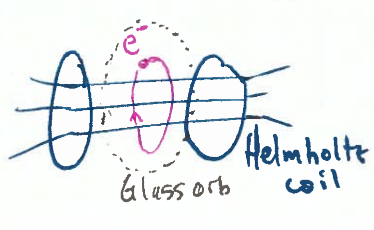
Radius of charge orbit: $\frac{mv^2}{r} = qvB \rightarrow \underline{r = \frac{mv}{qB}}$ by Lorentz

Time for charge to go around: $T = \frac{2\pi r}{v} \rightarrow \underline{T = \frac{2\pi m}{qB}}$ independent of velocity!

Consider magnetic field of $7.8\,e^{-10}$ T,
then $T = 46$ ns (by above)

Given potential difference of 100 V
then $q\Delta V = \frac{1}{2}mv^2 \rightarrow v \approx 5.9 \times 10^6$ m/s
Radius of electrons in this B-field is
$\approx r = 4.3$ cm

DEMO: Create uniform field w/ Helmholtz coil

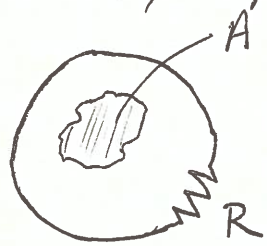


Place glass orbit w/ low pressure gas in field so some electrons can move in a circle

Faraday's Law:

$$\mathcal{E}_{induced} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$$

Stationary loop with changing Magnetic Field



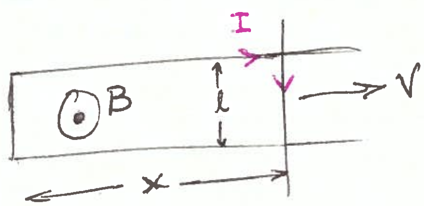
$$\Phi_B = AB$$

$$\frac{d\Phi_B}{dt} = A\frac{dB}{dt}$$

$$\mathcal{E}_{ind} = \left|A\frac{dB}{dt}\right|$$

$$I_{ind} = \mathcal{E}_{ind}/R$$

Constant Magnetic Field, changing geometry

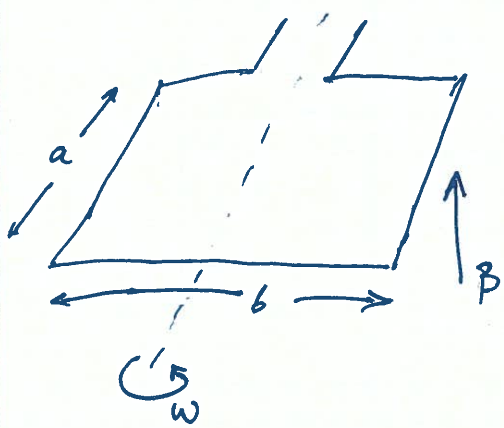


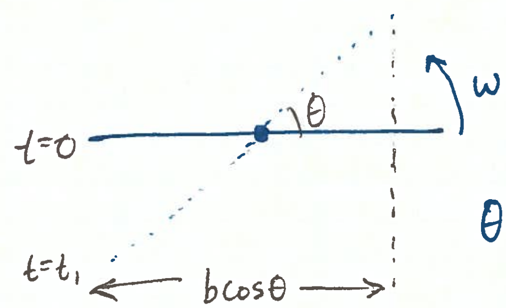
$$\Phi_B = B\cdot x l$$

$$\frac{d\Phi_B}{dt} = lBv$$

$$I_{ind} = \frac{lBv}{R}$$

Faraday Motor





Side View

$$\theta = \omega t$$

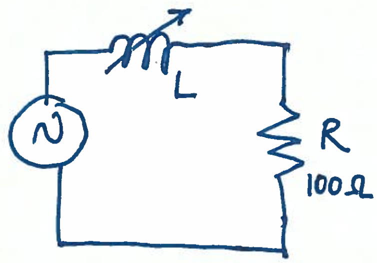
$$\Phi_B = B\, ab\cos\theta$$

$$\frac{d\Phi_B}{dt} = -\omega ab B\sin(\omega t) = \mathcal{E}_{ind}$$

$$I_{ind} = \frac{\mathcal{E}_{ind}}{R}$$

current is greater when motor spins faster!

RL Circuits



$V = V_0 \cos(\omega t)$

$\omega = 60\ Hz$

$V_0 = 100\ V$

[1] ENERGY DISSIPATION IN LIGHTBULB?

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\Phi_B \quad (\neq 0 \text{ don't use Kirchoff!})$$

$$I = I_{max} \cdot \cos(\omega t - \phi)$$

$$\boxed{I_{max} = \frac{V_0}{\sqrt{R^2 + (\omega L)^2}}} \; ; \; \tan\phi = \frac{\omega L}{R}$$

[a] $L = 0$, then $I_{max} = \frac{V_0}{R} = 1A$

Time Avg. Power $\langle I^2 R \rangle = \frac{1}{2} I_{max}^2 R = 50W$

[b] Increase L to $L = 300\ mH$

so $\omega L = 113\ \Omega$, $I_{max} = 670\ mA$

Time Avg Power $\langle I^2 R \rangle = 22\ W$

Q: Why use self inductor and not variable resistor???

↓

A: Variable Resistor can be used to change power dissipated by bulb too

BUT resistors lose energy in the form of heat and inductor stores energy in the field.